SECTION I, Part A

Time - 60 Minutes

Number of questions - 30

A CALCULATOR MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

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1. In the xy-plane, the graph of the parametric equations $x = 2 - 4t$ and $y = 6t + 5$, for $-2 \le t \le 5$, is a line segment with slope equal to

(A) -3

(B) $-\frac{3}{2}$

(C) $-\frac{2}{3}$

(D) $\frac{5}{2}$

2. $\lim_{x\to 1} \frac{\int_1^x e^t - 1\ dt}{x^2 - 1} =$

(A) $e - 1$

(B) e

(C) $\frac{e - 1}{2}$

(D) $\frac{e}{2}$

3. For what value of k is the function f continuous at $x = 4$?

$$f(x) = \begin{cases} \frac{\sqrt{3x + 4} - \sqrt{2x + 8}}{x - 4} & x \neq 4 \\ k & x = 4 \end{cases}$$

(A) 0

(B) $\frac{1}{16}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

4. What is the approximation of the value of $\ln 2$ obtained by using the fourth-degree Taylor polynomial about $x=0$ for $\ln(1+x)$?

(A) $1-\frac{1}{2}+\frac{1}{4}$

(B) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$

(C) $\frac{8}{3}-4$

(D) $1-\frac{1}{2}-\frac{1}{3}-\frac{1}{4}$

5. The interval of convergence of $\sum_{n=0}^{\infty}\frac{(x-2)^n}{5^n}$ is

(A) $-1<x<1$

(B) $-7<x\leq 7$

(C) $-3\leq x<7$

(D) $-3<x<7$

6. The length of the path described by the parametric equations $x = 2t^8$ and $y = 4t^4 + 2$, where $0 \le t \le 1$, is given by

(A) $16\int_0^1 \sqrt{t^{14} + t^6}\ dt$

(B) $\int_0^1 \sqrt{t^7 + t^3}\ dt$

(C) $4\int_0^1 \sqrt{t^{14} + t^6}\ dt$

(D) $16\int_0^1 \sqrt{t^9 + t^5}\ dt$

7. If $y = (\sqrt{x} + \sin x)^4$, then find y'.

(A) $4(\sqrt{x} + \sin x)^3$

(B) $4\left(\frac{1}{2\sqrt{x}} + \cos x\right)^3$

(C) $4\left(\frac{1}{2\sqrt{x}} + \cos x\right)$

(D) $4\left(\frac{1}{2\sqrt{x}} + \cos x\right)(\sqrt{x} + \sin x)^3$

8. If $f(x)=\frac{x}{\tan x}$, calculate $f'\left(\frac{\pi}{3}\right)$.

(A) $-\frac{\sqrt{3}}{3}$

(B) $\sqrt{3}$

(C) $\frac{3\sqrt{3}-4\pi}{9}$

(D) $\frac{\sqrt{3}(3+2\pi)}{9}$

9. Let f be a function such that $\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=9$. Which of the following must be true?

I. f is continuous at $x=3$.

II. f is differentiable at $x=3$.

III. f' is continuous at $x=3$.

(A) I only

(B) II only

(C) I and II

(D) II and III

10. What is the average value of $y=x^2\sqrt{3x^3+1}$ on the interval $[0,2]$?

(A) $\frac{124}{27}$

(B) $\frac{248}{27}$

(C) $\frac{124}{9}$

(D) $\frac{124}{3}$

11. Let f be the function given by $f(x) = x^3 - 75x + 10$. On which of the following intervals is the function f decreasing?

(A) $(-\infty, -5] \cup [5,\infty)$

(B) $[-5,5]$

(C) $[0,5]$

(D) $[0,5\sqrt{3}]$

12. $\int 3x^2 \ln 2x \ dx =$

(A) $\frac{x^3}{3} \ln 2x - \frac{x^3}{9} + C$

(B) $x^3 \ln 2x - \frac{x^4}{4} + C$

(C) $x^3 \ln 2x + \frac{x^3}{3} + C$

(D) $x^3 \ln 2x - \frac{x^3}{3} + C$

13. If a particle moves in the xy-plane such that at time $t > 0$ its position vector is $(\ln(t^3 + t^2), 5t^3 + 3)$, then the acceleration vector of the particle at $t = 1$ is

(A) $(1,30)$

(B) $\left(\frac{5}{2}, 15\right)$

(C) $\left(-\frac{9}{4}, 30\right)$

(D) $(1,15)$

t (hours)	5	8	11	15	19
R(t) (liters/hour)	6.5	5.0	4.0	3.5	2.0

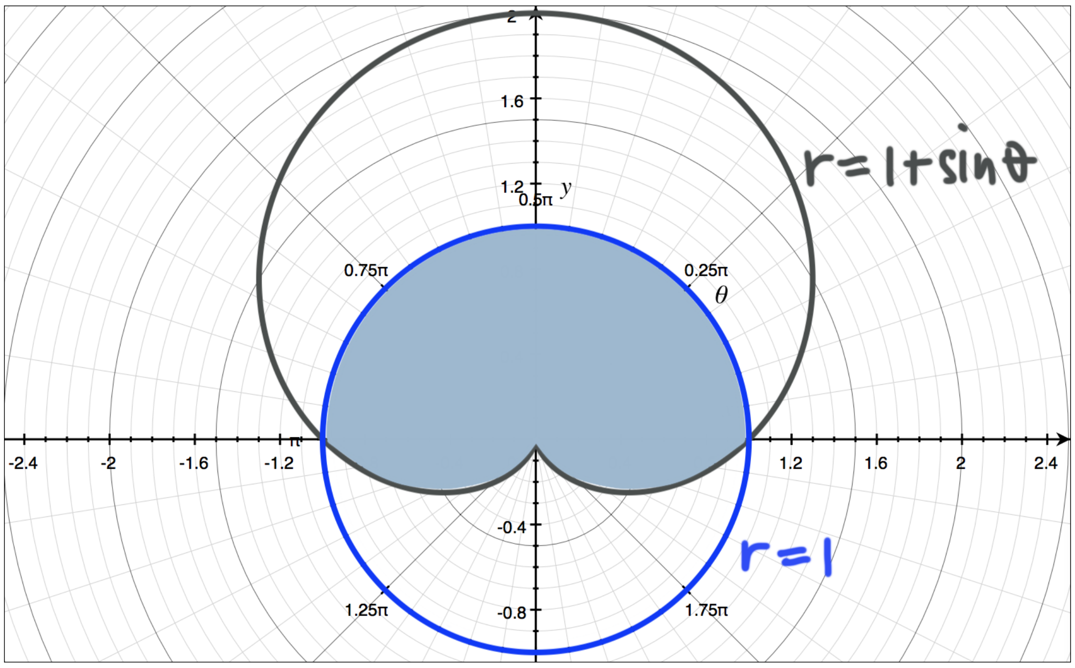
14. A tank contains 30 liters of water at time $t = 5$ hours. Water is being pumped into the tank at a rate $R(t)$ where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with four subintervals and data from the table, what is the approximation of the number of liters of water that are in the tank at time $t = 19$ hours?

(A) 49

(B) 73.5

(C) 79

(D) 88



15. The area of the region between the polar curves $r = 1 + \sin\theta$ and $r = 1$ is given by

(A) $\frac{1}{2}\int_0^{\pi} (1 - (1 + \sin\theta))^2 \ d\theta$

(B) $\frac{1}{2}\int_0^{\pi} 1 - (1 + \sin\theta)^2 \ d\theta$

(C) $\pi + \frac{1}{2}\int_{\pi}^{2\pi} 2\sin\theta + \sin^2\theta \ d\theta$

(D) $\pi - \frac{1}{2}\int_{\pi}^{2\pi} \sin\theta - \sin^2\theta \ d\theta$

16. Let f be the function defined by $f(x) = -\sqrt{|x|}$ for all values of x. Which of the following statements is true?

(A) f is differentiable at $x = 0$.

(B) f is not continuous at $x = 0$.

(C) $\lim_{x \to 0} f(x) \neq 0$

(D) f is continuous but not differentiable at $x = 0$.

17. The coefficient of x^3 in the Taylor series for e^{-2x} about $x = 0$ is

(A) $-\frac{8}{3}$

(B) $-\frac{4}{3}$

(C) $\frac{1}{6}$

(D) $\frac{4}{3}$

18. The function f is defined by $f(x) = \begin{cases} 2x+4 & x<1 \\ -3x+9 & x \geq 1 \end{cases}$. What is the value of $\int_0^3 f(x)\ dx$?

(A) -1

(B) 11

(C) 21

(D) 26

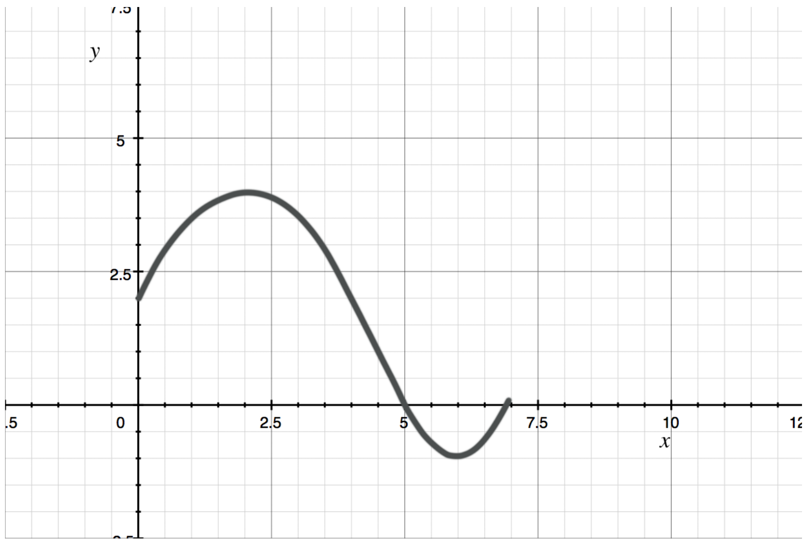
19. If $f(x) = 2x^3$ and $g(x) = 2x + 3$, then find the derivative of $f(g(x))$ at $x = 1$.

(A) 25

(B) 75

(C) 150

(D) 300



20. The graph of a differentiable function f is shown above. If $g(x) = \int_0^x f(t)\,dt$, which of the following statements is true?

(A) $g(5) < g'(5) < g''(5)$

(B) $g(5) < g''(5) < g'(5)$

(C) $g'(5) < g(5) < g''(5)$

(D) $g''(5) < g'(5) < g(5)$

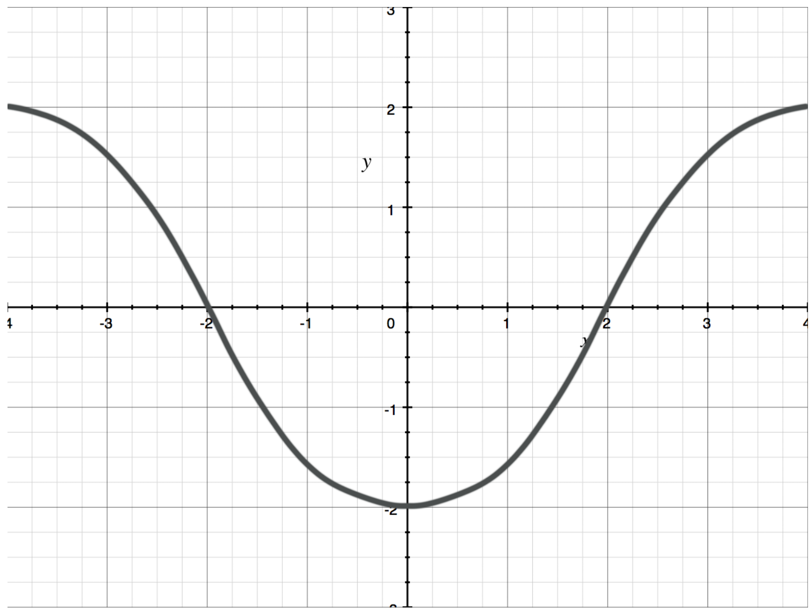
21. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^2e^{-3t}$. For what values of t the particle is at rest?

(A) 0 only

(B) $\frac{2}{3}$ only

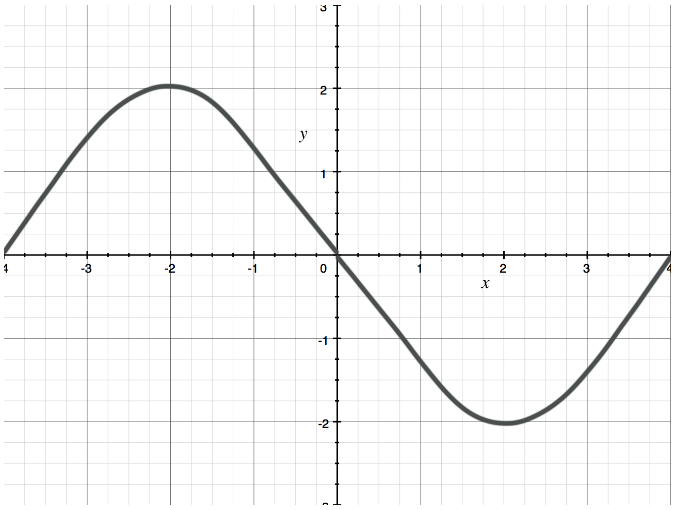
(C) 0 and $\frac{2}{3}$

(D) 1 only



22. The figure above shows the graph of f. If $f(x)=\int_0^x g(t)\ dt$, which of the following could be the graph of $y=g(x)$?

(A)



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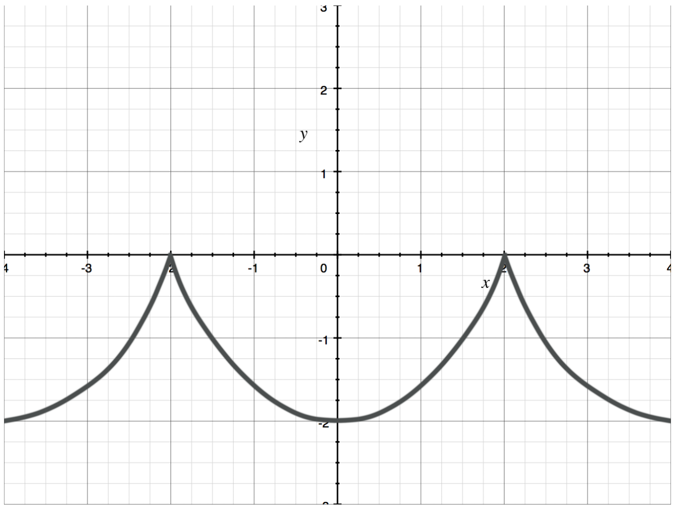
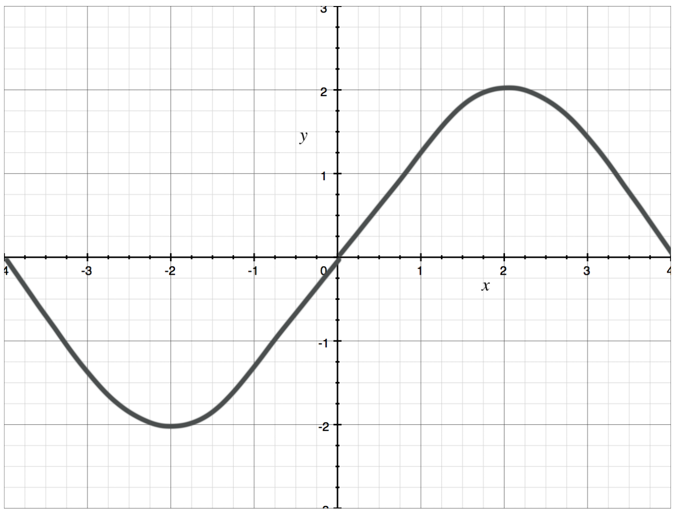
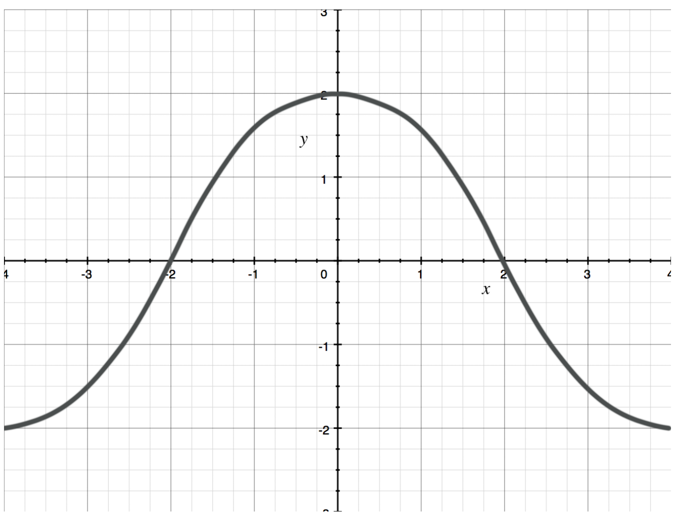


(C) 

(D) 

23. The function f is defined by $f(x) = \dfrac{x^2}{2x+1}$. What point (x, y) on the graph of f has the property that the line tangent to f at (x, y) has slope 0?

(A) $(0,0)$ only

(B) $(-1, -1)$ only

(C) $(0,0)$ and $(-1, -1)$

(D) $(-1, -2)$ only

24. Let $f(x)=x^5-1$ and let g be the inverse of f. Given that $f(1)=0$, what is the value of $g'(0)$?

(A) $-\frac{1}{4}$

(B) 0

(C) $\frac{1}{5}$

(D) 1

25. If the graph of $y=\frac{ax^2+b}{c-x^2}$ has a horizontal asymptote $y=5$ and vertical asymptotes $x=2$ and $x=-2$, then find $a+c$.

(A) -9

(B) -1

(C) 0

(D) 1

26. What is the absolute minimum of the function f given by $f(x) = xe^{2x}$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{2e}$

(C) $-\frac{e}{2}$

(D) 0

t	0	1
f(t)	3	7

27. Let $y = f(t)$ be a solution to the differential equation $\frac{dy}{dt} = k$, where k is a constant. The values of f for selected values of t are given in the table above. Which of the following is an expression for $f(t)$?

(A) $3e^{t\ln\frac{7}{3}}$

(B) $2t^2 + 3$

(C) $e^{t\ln 7} + 3$

(D) $4t + 3$

28. Let f be the function given by $f(x) = (x^2 + 1)e^{-kx}$, where k is a constant. For what value of k does g have critical points at $x = 1$?

(A) -1

(B) 0

(C) 1

(D) $\ln 1$

29. Which of the following is the solution to the differential equation $\frac{dy}{dx} = xe^{-y}$ with the initial condition $y(0) = 1$?

(A) $y = e^{\frac{x^2}{2}}$

(B) $y = \ln\left(\frac{x^2}{2}\right)$

(C) $y = \ln\left(\frac{x^2}{2} + \frac{1}{2}\right)$

(D) $y = e^{\frac{x^2}{2} + \frac{1}{2}}$

30. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \cos t + \sin t$. What is the acceleration of the particle at the point where the velocity is first equal to 0?

(A) $-\sqrt{2}$

(B) -1

(C) 0

(D) 1

END OF PART A, SECTION I

SECTION I, Part B

Time - 45 Minutes

Number of questions - 15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

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31. If $(3x^2y^2+2x)\frac{dy}{dx}=2y-2xy^3$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(-1,1)$?

(A) -4

(B) 0

(C) 4

(D) 30

32. If f is a vector-valued function defined by $f(t)=\left(\frac{1}{e^{-2t}}, \sin t\right)$, then $f''(t)=$

(A) $(4e^{2t}, -\sin t)$

(B) $4e^{2t}-\sin t$

(C) $2e^{2t}+\cos t$

(D) $(4e^{2t}, \sin t)$

33. The graph of the function represented by the Maclaurin series $1-2x+\frac{4x^2}{2!}-\frac{8x^3}{3!}+...+\frac{(-1)^n(2x)^n}{n!}$ intersects the graph of $y=x^2+4$ at which value of x?

(A) -2.159

(B) -0.761

(C) 0.761

(D) 2.159

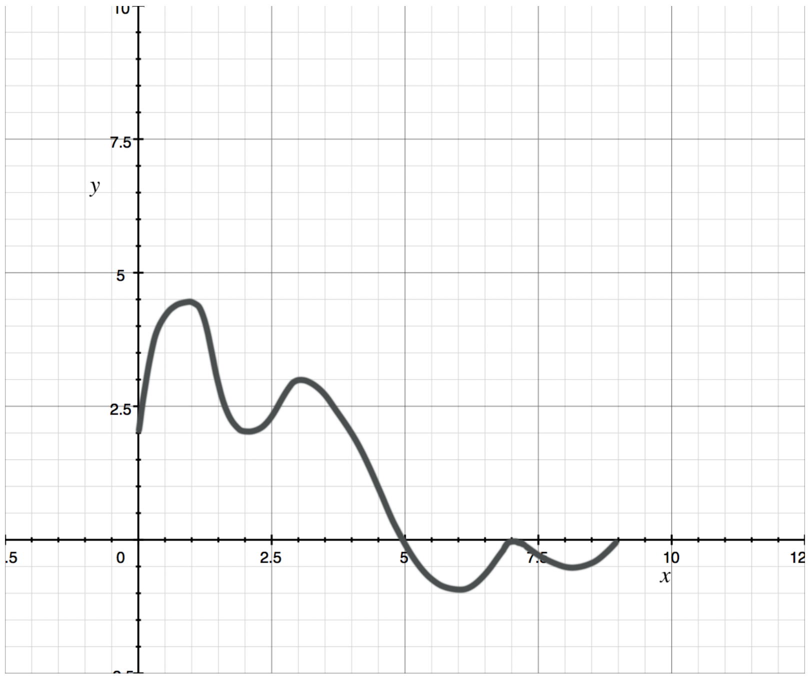
34. The graph of $y=3\ln(\sec x)$ crosses the x-axis at one point in the interval $[6,7]$. What is the slope of the graph at this point?

(A) -6.283

(B) -3.142

(C) 0

(D) 3



35. The function f is defined on the closed interval $[0,9]$. The graph of its derivative f' is shown above. Which of the following statements must be true?

I. f has six inflection points.

II. f has a relative minimum at $x = 5$.

III. The graph of f is concave down for $3 < x < 6$.

(A) I only

(B) II only

(C) III only

(D) I and III

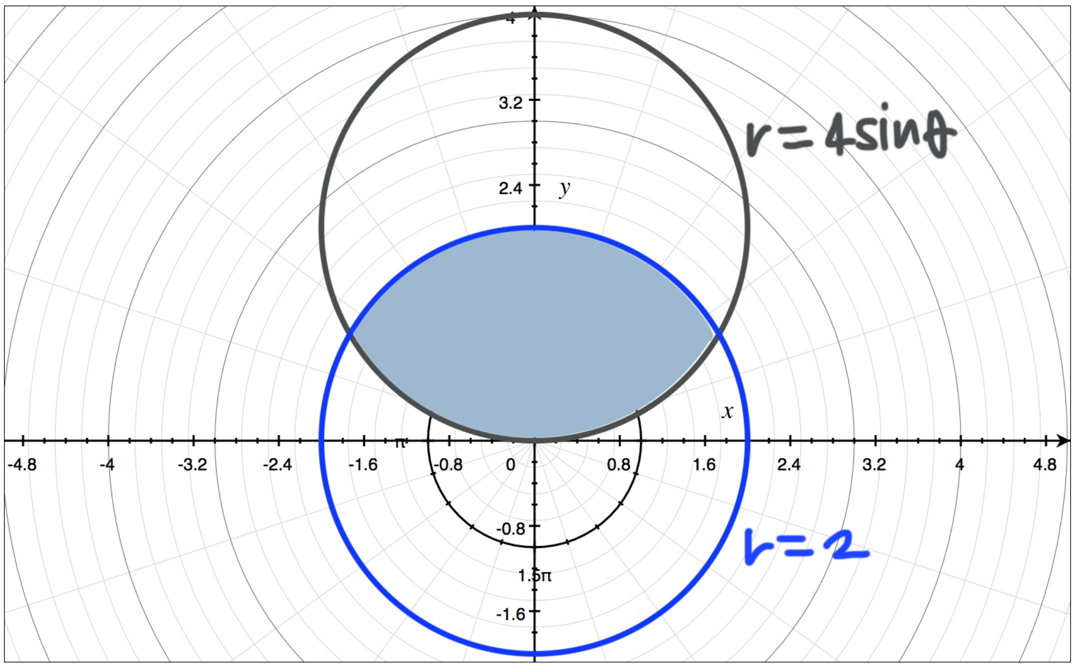
36. If $S_n = \left(\frac{(2+n^2)^{25}}{3^{n+2}}\right)\left(\frac{3^{n+1}}{(3+n^2)^{25}}\right)$, to what number does the sequence $\{S_n\}$ converge?

(A) $x > 0$

(B) $-\sqrt{3} < x < 0$ or $x > \sqrt{3}$

(C) $-3 < x < 0$ or $x > 3$

(D) $x > \sqrt{3}$



37. The figure above shows the graphs of the polar curves $r = 4\sin\theta$ and $r = 2$. What is the area of the shaded region?

(A) 1.369

(B) 4.914

(C) 2.739

(D) 7.653

38. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{\sqrt{n}}$ converges?

(A) $3 < x < 5$

(B) $3 \leq x < 5$

(C) $3 \leq x \leq 5$

(D) $-1 < x < 1$

39. If the base b of a triangle is increasing at a rate of 2 centimeters per minute while its height h is increasing at a rate of 2 centimeters per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.

(B) A is always decreasing.

(C) A is decreasing only when $h < -b$.

(D) A is decreasing only when $h > -b$.

40. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = t^2 + \cos t$. If the particle's velocity at $t = 0$ is -1, for what value of t will the velocity of the particle be zero?

(A) -0.88

(B) 0.71

(C) 0.78

(D) 0.88

41. Let f be a function such that $\int_5^{11} f(3x)\ dx = 15$. Which of the following must be true?

(A) $\int_{15}^{33} f(t)\ dt = 5$

(B) $\int_{15}^{33} f(t)\ dt = 45$

(C) $\int_5^{11} f(t)\ dt = 5$

(D) $\int_5^{11} f(t)\ dt = 45$

x	-3	0	2	5	7
f(x)	-3	2	7	1	4

42. Let f be a polynomial function with values of $f'(x)$ at selected values of x given in the table above. Which of the following must be true for $-3 < x < 7$?

(A) f has at least two critical points.

(B) f is concave down.

(C) f has two inflection points.

(D) The range of f is 7.

43. Find the area between the curves $y = 3x^2 + x - 2$ and $y = x + 3$.

(A) 4.304

(B) 6.545

(C) 6.873

(D) 8.607

44. Find the volume of the solid obtained by rotating the region enclosed by the curves $y=x^2+3$ and $y=x+3$ about the x-axis.

(A) 0.105

(B) 3.560

(C) 0.524

(D) 1.133

45. A railroad track and a road cross at right angles. An observer stands on the road 90 meters south of the crossing and watches an eastbound train traveling at 65 meters/second. At how many meters/second is the train moving away from the observer 8 seconds after it passes through the intersection?

(A) 0.985

(B) 38.057

(C) 64.048

(D) 65.967

END OF PART B, SECTION I

SECTION II, PART A

Time - 30 Minutes

Number of problems - 2

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS.

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1. Men enter a line for a cableway at a rate modeled by the function f such that

$$f(t) = \begin{cases} \frac{7}{1250}t^2\left(\frac{150-t}{t}\right)^6 & 0 \le t \le 150 \\ 0 & t > 150 \end{cases}$$

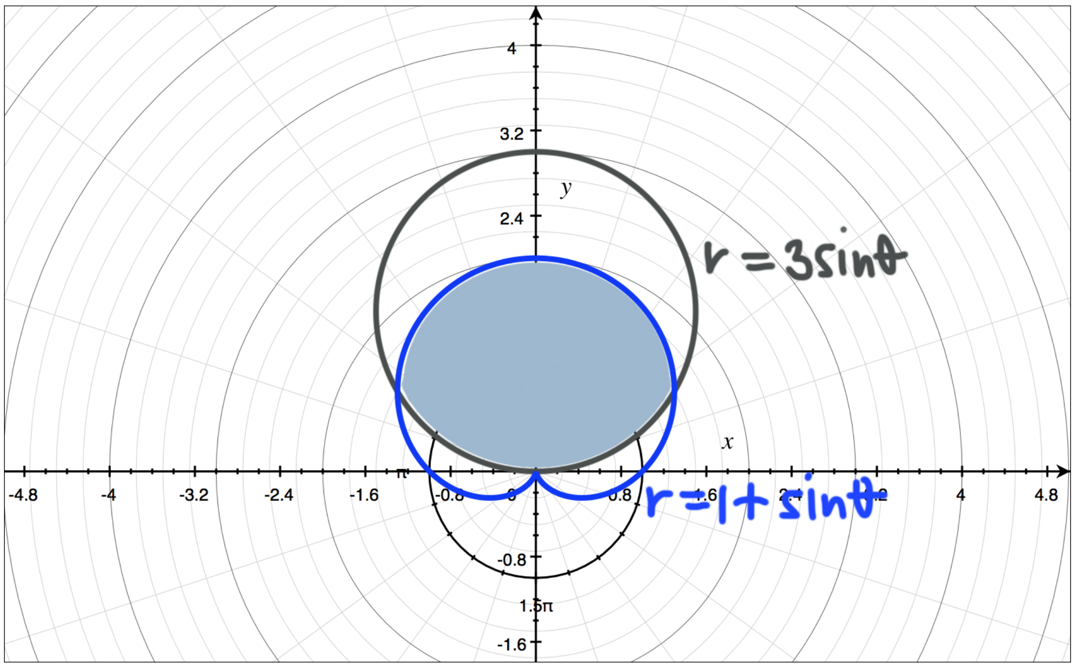
where $f(t)$ is measured in men per minute and t is measured in minutes. As men get on the cableway, they exit the line at a constant rate of 0.4 men per minute. There are 5 men in the line at time $t = 0$.

a. How many men enter the line for the cableway during the time interval $0 \le t \le 150$?

b. During the time interval $0 \le t \le 150$, there are always men in line for the cableway. How many men are in line at $t = 150$?

c. For $t > 150$, what is the first time t that there are no men in line for the cableway?

d. For $0 \le t \le 150$, at what time time t is the number of men in line a minimum? Find the number of men in line when the number of men in line is minimized? Round your answer to the nearest number of men.



2. The graph of the polar curves $r = 1 + \sin\theta$ and $r = 3\sin\theta$ is shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

a. Find the area of the shaded region.

b. Find the slope of the line tangent to the graph of $r = 3\sin\theta$ at $\theta = \frac{\pi}{3}$.

c. A particle moves along the polar curve $r = 1 + \sin\theta$ so that at time t seconds, $\theta = t^2$. Find the position vector in terms of t and the velocity vector at time $t = 2$.

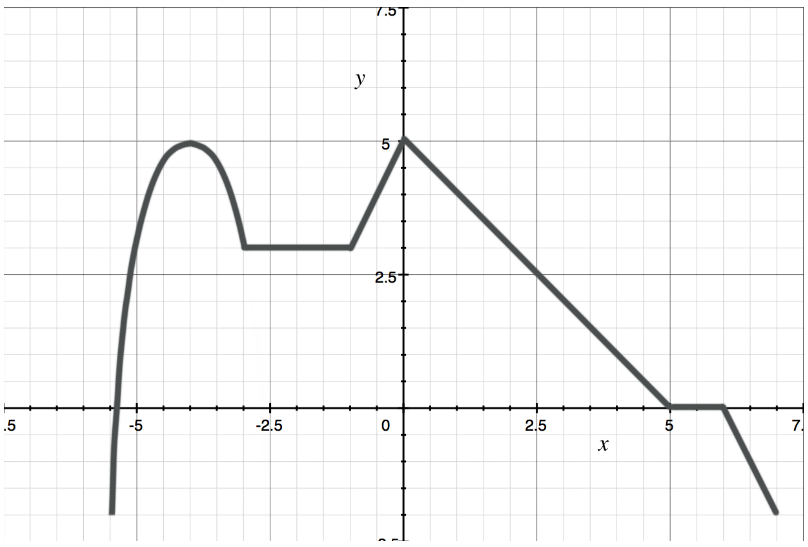
END OF PART A, SECTION II

SECTION II, PART B

Time - 60 **Minutes**

Number of problems - 4

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS.



3. The graph of the continuous function h, the derivative of the function f, is shown above. The function h is $h(x) = -2(x+4)^2 + 5$ for $-6 \le x \le -3$ and piecewise linear for $-3 \le x \le 7$.

a. If $f(-1) = 3$, what is the value of $f(7)$?

b. Evaluate $\int_{-6}^{-1} g(x)\ dx$.

c. For $-6 < x < 7$, on what open intervals, if any, is the graph of f both decreasing and concave down? Give a reason for your answer.

d. Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

Distance x (cm)	0	2	3	5	8	10
Temperature T(x) (°C)	100	90	81	73	60	51

4. A metallic wire of length 10 cm is heated at one of its ends. The table above gives selected values of the temperature of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

a. Determine $T'(9)$. Show your work.

b. Write an integral expansion in terms of $T(x)$ for the average temperature of the wire. Determine the average temperature of the wire using a trapezoidal sum with five subintervals shown by the data in the table.

c. Find $\int_0^{10} T'(x)\ dx$. What is the meaning of $\int_0^{10} T'(x)\ dx$ in terms of the temperature of the wire.

d. Explain why there must be at least one distance x, for $0 < x < 10$, such that $T'(x) = -4$.

5. The Maclaurin series for $\frac{1}{1-x}$ is given by $1 + x + x^2 + x^3 + \ldots + x^n + \ldots$. On its interval of convergence, this series converges to $\frac{1}{1-x}$. Let f be the function defined by $f(x) = \frac{x^2}{1+3x}$.

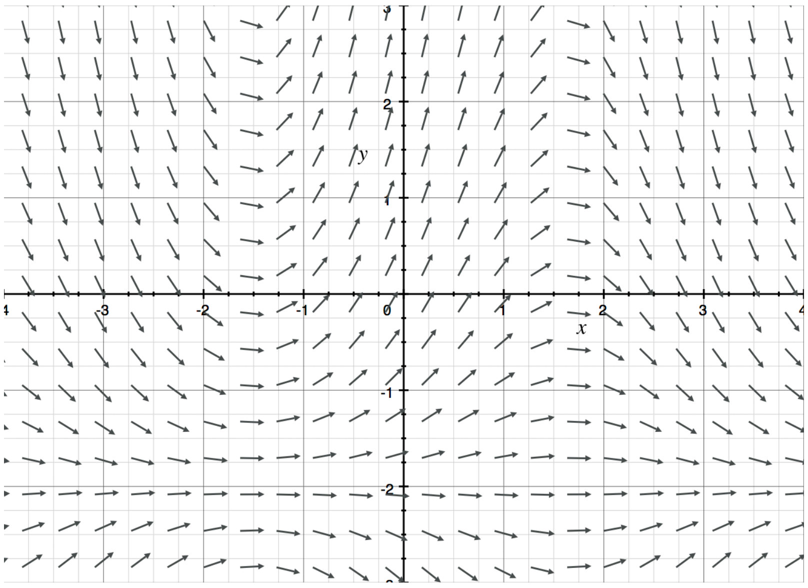
a. Write the first four nonzero terms and the general term of the Maclaurin series for f.

b. Determine the interval of convergence of the Maclaurin series for f. Show your work.

c. Let $P_n\left(\frac{1}{3}\right)$ represent the nth-degree Taylor polynomial for f about $x = 0$ evaluated at $x = \frac{1}{3}$. Use the alternating series error bound to show that $\left|P_4\left(\frac{1}{3}\right) - f\left(\frac{1}{3}\right)\right| < \frac{1}{2}$.

6. Consider the differential equation $\frac{dy}{dx} = (y + 2)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 0$. The function f is defined for all real numbers.

a. A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0,0)$.



b. Write an equation for the line tangent to the solution curve in part a at the point $(0,0)$. Use the equation to approximate $f(0.5)$.

c. Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 0$.

STOP

END OF EXAM